**Number:** Operations – Multiplication 2

**5. Always, Sometimes, Never**

This task encourages students to perform an investigation. They are presented with a hypothesis – *When you multiply, the product is greater than either of the two numbers being multiplied* – and are asked to investigate whether it is always true, sometimes true or never true.

**Focus Skills:**

* *Communicating and expressing*: Discuss and explain the processes used and the results of mathematical activities, problems and projects in an organised way.
* *Reasoning*: Make informal deductions.

**Teaching Points:**

* In Parts A, B and C, students are scaffolded through investigating three different scenarios. They are encouraged to experiment with one-digit numbers, multi-digit numbers, numbers that include decimals and ‘interesting’ numbers, and to perform multiplication calculations.
* Students should have experience multiplying a decimal number by a whole number, and a fraction by a whole number, before engaging with this task. Extend the task to finding examples of products of decimal number × decimal number and fraction × fraction calculations, if appropriate.
* The use of a calculator should be encouraged if performing calculations is limiting a student’s ability to investigate the statement.
* Calculators can be used to generate multiple examples quickly and could be combined with written calculations. You could say, for example: *After ten minutes working on Part A you may use a calculator to help you generate extra examples.*
* Encourage students to select a broad array of numbers as they perform their investigations.
* In response to the Maths Talk question, students should be able to draw on their examples from Parts A, B and C to show that it is sometimes true.

**Anticipated Student Responses**:

Students will find different examples for each part of the task. Some examples are shown below. These examples consider only positive numbers as students will not yet have experienced multiplication of negative numbers.

|  |  |  |
| --- | --- | --- |
| **Part A** | **Part B** | **Part C** |
| [number > 1] × [number > 1], e.g.:2 × 3 = 4 276 × 54 = 14,9046.325 × 9 = 59.9253.45 × 1.9 = 6.555 1$\frac{1}{2}$ × 6 = 9 | [number > 1] × [number ≤ 1], e.g.:3 × 0.5 = 1.5 26 × 0.3 = 7.8 1.3 × 0.2 = 0.263 × $\frac{3}{4}$ = 2$\frac{1}{4}$ | [number < 1] × [number ≤ 1], e.g.:0.3 × 0.2 = 0.06 0.125 × 0.6 = 0.075 $\frac{1}{10}$ × $\frac{3}{4}$ = 0.075$\frac{1}{2}$ × $\frac{1}{2}$ = $\frac{1}{4}$ |
| **Extension** |
| Students should follow a similar approach to the one they used for the main task to investigate the statement. They should conclude that the statement is sometimes true as an odd number × an odd number will always result in a product that is odd, but an even number × an odd number will always result in a product that is even. |

**Algebra:** Directed Numbers

**15. Working with Negatives**

This task encourages students to order positive and negative numbers. It subsequently asks them to complete operations, in the form of number sentences, using the numbers. The introduction of negative numbers in number sentences will be relatively new to many students.

**Focus Skills:**

* *Applying and problem-solving*: Analyse problems and plan an approach to solving them.
* *Integrating and connecting*: Understand the connections between mathematical procedures and the concepts they use.

**Teaching Points:**

* Some students will be very comfortable working with positive and negative numbers while others may not be. Number lines and calculators could be made available to students, as appropriate.
* Some students may benefit from having physical number cards with numbers matching those shown on p. 34, which they can manipulate to order and create number sentences.
* In the Maths Talk aspect of the lesson, students are asked to give example of when they would encounter negative numbers in real life. Examples include, elevators, thermometers, freezer door displays, digital kitchen scales (when you turn the scale on with a bowl on it and then remove the bowl, for example), bank account statements.
* There are multiple solutions to Part C. Students could be encouraged to find more than one solution. Use calculators to assist thinking, as necessary.

**Anticipated Student Responses**:

|  |
| --- |
| **Part A** |
|  |
| **Part B** |
| −13 + 3 = −10 −7 + −1 = −810 − 12 = −2 5 − −4 = 9 |
| **Part C** |
| Possible combinations of cards include: 12 + −8 + −4 + −13 + −2 = −15 −8 + −2 + −7 + 3 + −1 = −1512 + −10 + −8 + −7 + −2 = −15 |

**Shape and Space:** 3D Shapes

**19. The Platonic Solids**

This task focuses on the Platonic solids and Euler’s formula. Students construct 3D shapes, analyse the shapes and investigate patterns that emerge from the observations they make. Students are introduced to Euler’s formula and asked to investigate whether or not it is true for different shapes.

**Focus Skills:**

* *Communicating and expressing*:Discuss problems and carry out analyses.
* *Reasoning*: Make informal deductions.

**Teaching Points:**

* As students engage with this problem, they should have access to resources to help them construct the Platonic solids. 3D shape construction sets such as Polydrons and Magformers allow students to create these shapes easily. The shapes could also be constructed using toothpicks and clay. Paper nets can also be used, but additional time will need to be allocated to allow students to create the shapes.
* For Part A, students will need to devise a method of ensuring that they only count each face, vertex and edge once only. Placing a dot of Blue Tack on the counted item is one way of doing this.

As part of the Maths Talk aspect of the lesson, students may notice that, for example, there are always more edges than faces or vertices for all solids; ‘hexa’ implies 6 and the hexahedron has six faces. ‘Octa’ implies 8 and the octahedron has eight faces. ‘Tetra’ must imply 4 and ‘icosa’ must imply 12; the faces and the vertices combined are almost the same as the number of edges.

* For Part B, students will use Euler’s formula: faces + vertices − edges = 2. Faces refers to number of faces, vertices to number of vertices and edges to number of edges a polyhedron has.
* For Part C, if suitable materials are available, encourage students to make the icosahedron to test if their calculation is correct. If not search, for an ‘icosahedron interactive’ online.

**Anticipated Student Responses**:

|  |  |
| --- | --- |
| **Part A**  | **Part B** |
|

|  |  |  |  |
| --- | --- | --- | --- |
| **Shape**  | **Faces**  | **Vertices**  | **Edges**  |
| Tetrahedron  | 4 | 4 | 6 |
| Hexahedron  | 6 | 8 | 12 |
| Octahedron  | 8 | 6 | 12 |
| Dodecahedron  | 12 | 20 | 30 |

 | Students should do the following calculations to find that Euler’s formula is true for the four Platonic solids they looked at in Part A. Tetrahedron: 4 + 4 − 6 = 2Hexahedron: 6 + 8 − 12 = 2Octahedron: 8 + 6 − 12 = 2Dodecahedron: 12 + 20 − 30 = 2 |
| **Part C** |
| 30 edges |

**Measures:** Capacity and Volume

**25. Building Towers**

This task focuses on finding the volume of cuboids. It encourages reinforcement of the formula: volume of a cuboid = length × depth × height. This task also develops algebraic thinking as students are encouraged to search for patterns in the sequence and to determine what future terms in the sequence would look like. The extension activity links 3D shape as students are encouraged to create a net for one of the towers (a cuboid) and construct the tower.

**Focus Skills:**

* *Reasoning*: Search for and investigate mathematical patterns and relationships.
* *Applying and problem-solving*:Apply concepts and processes in a variety of contexts.

**Teaching Points:**

* This problem links Capacity (Volume) with Algebraic thinking and an understanding of 3D construction.
* The use of calculators, as the students engage with this problem, should be encouraged.
* If students find it difficult to visualise the first three towers in the sequence, the towers could be constructed using interlocking cubes – allowing each cube to represent 1 cm cubed.
* Encourage students to examine the information they have, i.e. length, depth, height and volume and to search for patterns and relationships. This is the focus of Maths Talk in this task and could be explored further during whole-class discussion. Students may note that: each dimension of the tower is doubling as you move from one tower to the next; the volume of each tower can be found by multiplying the volume of the previous tower by 8; the height is always double the length/depth; The length and depth are always the same.

**Anticipated Student Responses**:

|  |
| --- |
| **Part A**  |
| Tower 1: 1 cm × 1 cm × 2 cm = 2 cm3Tower 2: 2 cm × 2 cm × 4 cm = 16 cm3Tower 3: 4 cm × 4 cm × 8 cm = 128 cm3 |
| **Part B** |
| Tower 3 = 128 cm3 so Tower 4 = 128 × 8 = 1,024 cm3Tower 4 = 1,024 cm3 so Tower 5 = 1,024 cm3 × 8 = 8,192 cm3 Alternatively, students could find the dimensions of each of the new towers and multiply, i.e. Tower 4 = 4 cm × 4 cm × 8 cm and Tower 5 = 8 cm × 8 cm × 16 cm  |
| **Part C** |
| 128 cm × 128 cm × 256 cm = 4,194,304 cm3 |

**Anticipated Student Responses:**

|  |
| --- |
| **Part A**  |
| Add parents’ heights: 177 cm + 163 cm = 340 cmAdd 13 cm: 340 cm + 13 cm = 353 cm Divide by 2: 353 cm ÷ 2 = 176·5 cm |
| **Part B** |
| Add parents’ heights: 177 cm + 163 cm = 340 cmSubtract 13 cm: 340 cm – 13 cm = 327 cm Divide by 2: 327 cm ÷ 2 = 163·5 cm |
| **Part C** |
| Students could work backwards using Mam’s height and inverse operation to multiply by 2: 163 cm × 2 = 326 cm Add 13 cm (for a girl) rather than subtract: 326 cm + 13 cm = 339 cm So the combined height of Gran and Grandad is 339 cm, but we know Grandad is 15 cm taller than Gran. Divide their combined height by 2: 339 cm $÷$ 2 = 169·5 cmAdd half of the 15 cm difference to find Grandad’s height: 169·5 cm + 7·5 cm = 177 cm Subtract half of the 15 cm difference to find Gran’s height: 169·5 cm – 7·5 cm = 162 cm |

**Data:** Chance

**30. Roll the Dice**

This task focuses on rolling two dice, combining the total of the numbers rolled and exploring the different possible outcomes.

**Focus Skills:**

* *Reasoning*: Search for and investigate mathematical patterns and relationships.
* *Integrating and connecting*: Represent mathematical ideas and processes in different models.

**Teaching Points:**

* Students complete the task in pairs or in groups. Encourage groups to compare findings.
* If there are many dice rolling on tables, it can create a lot of noise. Consider using foam dice or allowing students to conduct the experiment on the floor.
* Part A is presented as a hands-on activity, to help students become familiar with the idea of the lesson. Subsequently, they are encouraged to explore all possible outcomes, through the use of a Y-table and explore the connections between the hands-on activity and the table they create.
* As students complete the task in Part A, ask questions such as – *What do you notice?*; *What totals are you getting most frequently?*; *Is anything unusual or surprising?*; *What totals are occurring least frequently?*
* Whole-class discussion should take place after the students complete the table. Focus on what the table tells us, i.e. that the lowest possible total is 2, the highest possible total is 12, the most likely outcome is 7 with a ‘6 in 36’ or ‘1 in 6’ chance.
* Encourage students to use the language of probability. Sentences such as ‘1 in 36’ and words such as certain, unlikely, likely and impossible should be encouraged.

**Anticipated Student Responses**:

|  |
| --- |
| **Part A** |
| Results will vary.  |
| **Part B** |
| Students can make different suggestions here. They are being encouraged to look back at the results in Part A and consider what is likely or unlikely. They might say it is more likely to get a 6, a 7 or an 8 as there are three combinations of rolls for each, e.g. for a total of 8: 2 + 6, 3 + 5 and 4 + 4 |
| **Part C** |
| Students can make different suggestions here. They are being encouraged to look back at the results in Part A and consider what is likely or unlikely. They might say, it is unlikely to get a total of 2 or 3 as there is only one combination of rolls each, 1 + 1 and 2 + 1.  |
| **Part D** |
|  |
| **Part E** |
| Possible things the students could notice:* The numbers are all the same on the diagonal.
* The number 7 appears the most times.
* 12 is the greater total you can make from rolling two dice, 2 is the smallest.
* There is a ‘1 in 36’ chance of getting a 2 and a ‘1 in 36’ chance of getting a 12.
* There is a ‘2 in 36’ chance of getting a 2 or a 12.
* It is impossible (zero chance) to make a total of 1 or a total greater than 12.
* The most likely outcome is a 7, with a ‘6 in 36’ or a ‘1 in 6’ chance.
* The chance of getting a 6 or an 8 is the same, ‘5 in 36’ in each case.

As students compare their hands-on activity in Part A with their table in Part D, they may notice the following (depending on their outcomes in Part A) for example:* The same numbers appear in both tables.
* There is one 12 in my table in Part D. I didn’t get 12 in Part A. It must be unlikely to get an answer of 12.
* I got the number 2 more than once in my table. While it’s unlikely to get a score of 2 often, it is not impossible.
* I have lots of 6s, 7s and 8s in the table in Part A too.
 |